ON PLASTIC CAVITATION

(O PLASTICHESKOM RAZRYKHLENII)

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1. The limiting state of a granular medium is defined by the equality

$$g_1 = |\tau_m| + \alpha\sigma_n = \tfrac{1}{2}(\sigma_1 - \sigma_3) + \tfrac{1}{2}\alpha(\sigma_1 + \sigma_3) = \alpha S = \tau_* \qquad (1.1)$$

in which S is the temporary resistance of the medium in tension; α is its coefficient of internal friction; τ_* is the limiting resistance of the medium to pure shear and σ_1, σ_2, σ_3 are the principal stresses (it being assumed that $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$). Prandtl [1] and Guest [2 and 3] have proposed (1.1) as a yield criterion for solid bodies, considering these as granular media with a high cohesion between particles. An analogous criterion has been proposed by Deriagin [19] on the basis of a physical analysis. Evidenly S should be taken to be the theoretical and not the actual strength of the material of the body, since the latter depends on local defects, whereas in the context of Formula (1.1) it must include the cohesive stress averaged over the whole shear plane.

Expression (1.1) is a refinement of St.Venant's criterion in that it takes into account the effect of normal stress on the value of the critical tangential stress.

A related criterion would be that of Schleicher [4]

$$g_2 = \sigma_i + \beta\sigma = \sqrt{2}\,\tau_* \qquad (\beta = \mathrm{const}) \qquad (1.2)$$

Where

$$\sigma_i = \frac{1}{\sqrt{3}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sqrt{\sigma_{ij}'\sigma_{ij}'} \qquad (1.3)$$

(σ_{ij}' are the components of the deviator of the stress tensor)

$$\sigma = \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \tfrac{1}{3}\sigma_{ii} \qquad (1.4)$$

Expression (1.2) is a refinement of Mises' criterion in that it takes into account the effect of the mean normal stress on the critical value of the intensity of shear stress (or, what amounts to the same thing, of mean shear stress [6]).

2. In generalizing criteria (1.1) and (1.2) to strain-hardening materials two extreme hypotheses are possible.

a) The hardening is governed by an increase in the coefficient of internal friction.

b) hardening depends on the internal elastic forces of an intergranular and inter-block nature.

If (a) is valid then the boundary of the region of elastic strains (determined without taking into account the effect of σ or σ_n) expands in all directions under plastic deformation, and if (b) is valid, this region (again determined without taking into account the effect of σ or σ_n) is displaced as a rigid body [7].

In actual fact both these effects exist and, as experiment shows, (see, for example, [8]) initially (with plastic strains not exceeding 1 – 2%) the effect of boundary translation predominates but thereafter the process is mainly one of expansion. The same conclusion is reached from the results of experimental studies of the heat generated during plastic deformation.

It has been established that part of the work done in plastic deformation is not converted into heat, which indicates the accumulation within the body of latent elastic energy. The ratio of this part of the work to the total work done in the plastic deformation decreases monotonously with increase in the latter [9]. It follows that the part played by elastic microstresses in the hardening process becomes less significant and gives way to the effect of the increase in frictional forces.

These phenomena may be explained as follows: polycristalline bodies, being microscopically (and supermicroscopically) heterogeneous and anisotropic (on account of their granular structure and in view of the structural defects in each individual grain) constitutes (from the point of view of structural mechanics) a statically indeterminate system with a huge number of elements. As the loading increases the elements of this system enter the plastic range not simultaneously but gradually, which macroscopically is observed as a monotonous increase in the coefficient of friction. In addition, as plastic deformation develops, elastic interactions are set up between the element of the system which are interpreted macroscopically as a hardening of the material with increase in load and a softening of the material under plastic deformation in the opposite direction (hence the Bauschinger effect).

Special mention should be made of alternating plastic deformation which we shall now discuss in some detail. The work done in such deformation increases with the number of cycles n and is approximately proportional to this number, the magnitudes of the plastic strains (or stresses) lying within certain specified limits. It has been discovered ([10], [11] and others) that the plastic hysteresis loop after an initial stage usually becomes steady: the material, so to speak, adapts itself to the cyclic loading. In fact this means that as the number of cycles increases the magnitude of the coefficient of internal friction becomes stabilized and thereafter the shape and size of the hysteresis loop is determined only by microelastic effects. It is true that one can find references [12 and 13] which indicate that stability of the plastic hysteresis loop under cyclic loading is not always achieved; as the number of cycles increases the loop either narrows monotonously or it widens monotonously. The first case corresponds to a monotonous increase in the coefficient of internal friction (with increase in the number of cycles) and the second to a monotonous decrease. Nevertheless, in cyclic loading microelastic effects undoubtedly predominate over the effects of change in internal friction, particularly when the loop is narrow.

Consequently, as a fundamental hypothesis, we shall assume that the coefficient of internal friction is constant. However, in order to be able to compare the results which follow from both hypotheses the other extreme case will be studied, in which hardening is attributed wholly to an increase in the coefficient of internal friction.

3. If we take the coefficient of internal friction to be constant and postulate that the relation between plastic strains and the macroscopic tensor s_{ij}, which defines the elastic microstresses [14 and 7], is linear, then the criterion (1.1) can be generalized as follows:

$$g_1^* = \tfrac{1}{2}(\sigma_1 - \sigma_3) - \tfrac{1}{2}G^*(\varepsilon_1^p - \varepsilon_3^p) + \tfrac{1}{2}\alpha(\sigma_1 + \sigma_3) = \alpha S = \tau_*^{(0)} \quad (3.1)$$

Here ε_1^p, ε_2^p, ε_3^p are the principal components of the plastic strain tensor (it being assumed that $\varepsilon_1^p \geqslant \varepsilon_2^p \geqslant \varepsilon_3^p$), G^* is the strain hardening modulus in shear, $\tau_*^{(0)}$ is the initial plastic resistance of the material to pure shear (i.e. the resistance to shear when $\varepsilon^p{}_{ij} = 0$).

Similarly, the criterion (1.2) can be generalized to the case of an ideal Bauschinger effect [7].

$$g_2^* = \sigma_i^\circ + \beta\sigma = \sqrt{2}\ \tau_*^{(0)} \quad (3.2)$$

Here

$$\sigma_i^\circ = \sqrt{\sigma_{ij}^{\circ\prime}\sigma_{ij}^{\circ\prime}} = \frac{1}{\sqrt{3}}\sqrt{(\sigma_1^\circ - \sigma_2^\circ)^2 + (\sigma_2^\circ - \sigma_3^\circ)^2 + (\sigma_3^\circ - \sigma_1^\circ)^2} \quad (3.3)$$

$$\sigma_{ij}^\circ = \sigma_{ij} - s_{ij}, \qquad \sigma_{ij}^{\circ\prime} = \sigma_{ij}^\circ - \tfrac{1}{3}\sigma_{ii}^\circ\delta_{ij} \quad (3.4)$$

$$s_{ij} = 2G^*\varepsilon_{ij}^p \quad (3.5)$$

The quantities $\tau_*^{(0)}, \alpha, \beta$ and G^* in (3.1) and (3.2) are assumed to be constant. We apply the associated flow law

$$d\varepsilon_{ij}^p = h\frac{\partial F}{\partial \sigma_{ij}}\, dF \quad (3.6)$$

As a loading criterion we take $F = g_1^*$ (3.1). In this way we obtain the following relations between stresses and plastic strains:

$$(d\varepsilon^p)_1 = \tfrac{1}{2}(1+\alpha)\, hdg_1^*, \qquad (d\varepsilon^p)_2 = 0, \qquad (d\varepsilon^p)_3 = -\tfrac{1}{2}(1-\alpha)\, hdg_1^* \quad (3.7)$$

Here $(d\varepsilon^p)_j$ are the principal values of the tensor of plastic deformation increments $d\varepsilon^p{}_{ij}$. It follows from (3.7) that

$$de^p = (d\varepsilon^p)_1 + (d\varepsilon^p)_2 + (d\varepsilon^p)_3 = \alpha h dg_1^* = \alpha\,[(d\varepsilon^p)_1 - (d\varepsilon^p)_3] > 0 \quad (3.8)$$

This shows, therefore, that if we take a strain-hardening law in the form (3.1) it follows from the associated flow law that any plastic deformation must be accompanied by a residual monotonous increase in volume, which must be interpreted physically as the formation within the body of microscopic holes, i.e. as plastic cavitation.

If we now take (3.2) as the strain-hardening criterion and substitute this expression in the associated flow law (3.6), we obtain

$$d\varepsilon_{ij}^p = \left[\frac{\sigma_{ij}^{\circ\prime}}{\sigma_i^\circ} + \frac{1}{3}\beta\delta_{ij}\right] hdg_2^* \quad (3.9)$$

Thus the plastic deformation may be sub-divided into a deviator part

$$d\partial_{ij}^{p} = \frac{\overset{\circ\prime}{\sigma_{ij}}}{\sigma_i^{\circ}}\, hdg_2^* \tag{3.10}$$

and an all-round residual change in volume

$$de^p = d\varepsilon_{ii}^{p} = \beta hdg_2^* \tag{3.11}$$

Squaring (3.10) (in the scalar sence), we obtain

$$d\partial^p = \sqrt{d\partial_{ij}^{p}\, d\partial_{ij}^{p}} = hdg_2^* > 0 \tag{3.12}$$

where $d\partial^p$ is the differential of the arc of the deviator "path" of plastic deformation. Also,

$$de^p = \beta d\partial^p \quad \text{or} \quad e^p = \beta L > 0 \qquad \left(L = \int d\partial^p\right) \tag{3.13}$$

Here L is the length of the plastic deformation "path".

It is obvious that both the hardening laws (3.1) and (3.2) considered above lead to the conclusion that plastic deformation must be accompanied by a residual increase in volume (plastic cavitation). The difference in these two laws, however, is that according to (3.1) and (3.6) the additional plastic strains which result from taking σ_n into account in the yield criterion reduce to plane strain (an all-round expansion in the shear plane) whereas according to (3.2) and (3.6) the additional plastic strains resulting from taking into account the mean normal stress σ in the yield criterion reduce to an all-round (three-dimensional) expansion. Which of the two variants of the theory is closer to the truth must be established experimentally.

4. In order to complete the investigation we shall in addition derive formulas which correspond to the assumptio that hardening is governed by an increase in the coefficient of internal friction. Here we must take τ_* in (1.1) and (1.2) to be a function of the plastic strains which varies according to the hardening law. This means that the coefficients α and β will also be variable. As a first approximation we can estimate α by discarding the second term of the left-hand side of (1.1). Then

$$\alpha \approx \frac{\sigma_1 - \sigma_3}{2S} \tag{4.1}$$

The theoretical resistance of the material in tension should be considered as proportional to Young's modulus.

$$S = \frac{1}{k} E$$

where k is a non-dimensional constant of the order of 10 (see, for example, [15] page 19).

Thus

$$\alpha \approx k \frac{\sigma_1 - \sigma_3}{2E} \tag{4.2}$$

Substituting (4.2) into (1.1) we find that

$$g_1 = \frac{1}{2}(\sigma_1 - \sigma_3) + \frac{k}{4E}(\sigma_1^2 - \sigma_3^2) = \tau_* \tag{4.3}$$

Substituting (4.3) into the associated flow law we obtain the following formulas for the principal values of the tensor of plastic strain increments:

$$(d\varepsilon^p)_1 = \frac{1}{2}\left(1 + \frac{k}{E}\sigma_1\right) hdg_1, \; (d\varepsilon^p)_2 = 0, \; (d\varepsilon^p)_3 = -\frac{1}{2}\left(1 + \frac{k}{E}\sigma_3\right) hdg_1 \tag{4.4}$$

It follows from this that

$$de^p = (d\varepsilon^p)_1 + (d\varepsilon^p)_2 + (d\varepsilon^p)_3 = -\frac{k}{E}[\sigma_3(d\varepsilon^p)_1 + \sigma_1(d\varepsilon^p)_3] = \\ = \frac{2k}{E}[\tau_m d\gamma^p + \sigma_n d\varepsilon_n{}^p] = \frac{2k}{E}(dA_\tau + dA_n) \qquad (4.5)$$

Here A_τ is the work done by the maximum shear stresses and A_n is the work done by the normal stresses σ_n in the plastic deformation,

$$d\gamma^p = \tfrac{1}{2}[(d\varepsilon^p)_1 - (d\varepsilon^p)_3], \qquad d\varepsilon_n{}^p = \tfrac{1}{2}[(d\varepsilon^p)_1 + (d\varepsilon^p)_3] \qquad (4.6)$$

Also,

$$e^p = \frac{2k}{E}(A_\tau + A_n) > 0 \qquad (4.7)$$

We shall consider now the case when (1.2) is taken as the strain-hardening condition. Assuming the coefficient β to be variable, we can determine its value (to a first approximation) from the equality

$$\sigma_i \approx \frac{1}{3}\beta S, \qquad \beta = \frac{3\sigma_i}{S} = \frac{3k_1\sigma_i}{E} \qquad \left(S = \frac{1}{k_1}E\right) \qquad (4.8)$$

Here S is the theoretical strength in all-round tension.

Then

$$g_2 = \left(1 + k_1\frac{\sigma}{E}\right)\sigma_i = \tau_* \qquad (4.9)$$

Substituting (4.9) in the associated flow law, we find that

$$d\varepsilon_{ij}^p = \left[\frac{\sigma_{ij}'}{\sigma_i}\left(1 + \frac{k_1}{E}\sigma\right) + \frac{k_1}{E}\sigma_i\delta_{ij}\right]hdg_2 \qquad (4.10)$$

Thus the increment in the deviator of the plastic strain tensor is

$$d\partial_{ij}^p = \frac{\sigma_{ij}'}{\sigma_i}\left(1 + \frac{k_1}{E}\sigma\right)hdg_2 \qquad (4.11)$$

The second term in the square brackets in (4.10) corresponds to an all-round residual change in volume

$$de^p = d\varepsilon_{ii}^p = \frac{3k_1}{E}\sigma_i hdg_2 \qquad (4.12)$$

Squaring both sides of equality (4.11) (in a scalar sense), we obtain

$$\sqrt{d\partial_{ij}^p d\partial_{ij}^p} = d\partial^p = \left(1 + k_1\frac{\sigma}{E}\right)hdg_2 \qquad (4.13)$$

It follows from (4.12) and (4.13) that

$$de^p = \frac{3k_1}{E}\frac{1}{1 + \sigma k_1/E}\sigma_i d\partial^p \approx \frac{3k_1}{E}\sigma_i d\partial^p \qquad (4.14)$$

Whence

$$e^p = \frac{3k_1}{E}\int \sigma_i d\partial^p = \frac{3k_1}{E}A > 0 \qquad (4.15)$$

We see then that the two variants of the theory considered in this section (based on the assumption that the hardening effect must be ascribed to an increase in frictional forces) enable us to conclude that any plastic deformation must be accompanied by a residual increase in volume the magnitude of which is found to be proportional to the work done in the plastic deformation. An analogous conclusion is reached in the variants of the theory considered in the preceding Section (based on the assumpion that the hardening effect must be ascribed to microelastic forces) with, however, the quantitative difference that the residual increase in volume proves to be proportional not to the work done in the deformation, but to the length of the path of the plastic deformation.

5. The yield criteria (1.1) and (1.2) which take into account the effect of normal stresses on the resistance to plastic deformation were proposed a long time ago, although nowadays they are not used, and even mention of them is seldom encountered in the literature. This is explained by the fact that experiment shows only a slight effect of both σ and σ_n on the way plastic deformation arises and develops. In general this is in complete agreement with the estimate given above for the coefficient of friction α, which, according to (3.1), is

$$\alpha = \frac{\tau_*^{(0)}}{S} = k\frac{\tau_*^{(0)}}{E} \qquad (5.1)$$

(where k is of the order of 10), from which it follows that α (and consequently β as well) must be of the order of 0.01. As a rule this is precisely the order of the corrections made to the value of plastic strains when σ and σ_n are taken into account in the loading criteria, i.e. these corrections usually lie beyond the limits of accuracy of the formulation of the phenomenological theory of plasticity. However, there is a case (of considerable practical interest) in which these corrections are significant and comparable with terms of the basic order. This is the case of cyclic loading when the plastic strains oscillate between certain maximum and minimum values. When the cycle is symmetrical the path of plastic deformation is given by Formula

$$L = 2nf \qquad \left(f = \int d\vartheta^p\right) \qquad (5.2)$$

Here n is the number of cycles and the integration is carried out over one half-cycle. L increases proportionally to the number of cycles and for a sufficiently large n can reach a significant order inspite of the smallness of f.

Consequently the residual plastic change of volume as given by Formula (3.13) can become significant inspite of the smallness of the coefficient β. We arrive at analogous conclusions from the other variants of the theory considered above.

Thus, cyclic loading is a case which is particularly suitable for illustrating the corrections to be applied to the theory of plasticity when σ and σ_n are taken into account in the yield criteria, since the basic terms in the solution in this case are all the time bounded by definite limits, whereas the correction terms increase monotonously proportionally to the number of cycles. In using these results, however, it must be borne in mind that the formulas which were taken as the starting point refer to the case of quasi-static isotermal deformation. Therefore in the foregoing discussion of cyclic loading we must assume that the loading takes place sufficiently slowly. The possibility of applying the results to the case of rapidly varying loads requires a special investigation altough the qualitative aspect of the phenomenon would evidently still hold good.

6. The results we have summarized are based on two hypotheses.

a) The associated yield law is valid to an extremely high degree.

b) The influence of σ or σ_n on the yield boundary, even to a first approximation, can be taken into account by the criteria (1.1) and (1.2). These assumptions are open to question and the results obtained can be assessed only by experiment. At the request of the author Ia.S. Sidorin and O.G. Rybakina carried out some experiments on three tubular specimens of aluminium alloy subjected to alternating torsion. The dimensions of the specimens were as follows: external diameter d = 30mm, the length of the working part

l = 120mm and the thickness h = 2mm . The material of the specimens was annealed in order to eliminate initial anisotropy. The width of the plastic hysteresis loop in all experiments was taken as 1% (converting from shear to tension-compression). No restraint was imposed on the longitudinal deformation of the specimen. Small peripheral notches were cut on the specimens and the change in distance between these notches was measured during the tests. The change in the diameter of the specimens was also measured. The devices used measured the longitudinal extensions to an accuracy of 10^{-3} % and the

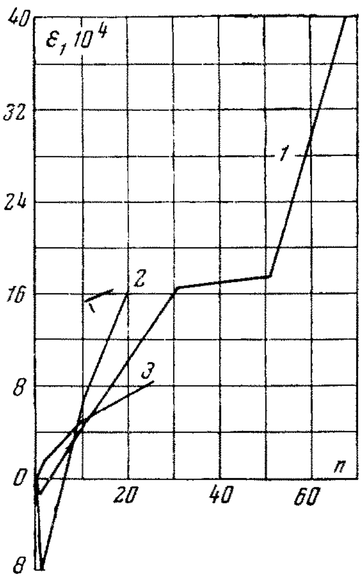

Fig. 1

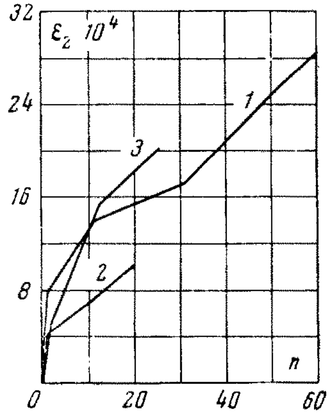

Fig. 2

lateral extensions to an accuracy of 2×10^{-2}% , which represented not more than 5% referred to the maximum extensions observed during the experiments. Consequently, the results obtained cannot be attributed to measuring errors. The results are shown in Figs. 1 and 2 from which it can be seen that during the first few cycles the curves of change in length and diameter of the specimens are of a complex and irregular nature, which evidently corresponds to a transistory phase — the period in which the material adapts itself to the cyclic loading. This is followed by a monotonous increase in both the length and the diameter of the specimens, this increase being approximately proportional to the number of cycles. The increments in the longitudinal and lateral strains over one cycle were found to be of the same order. Specimen 1 was subjected to 69 cycles; specimens 2 and 3 failed after 21 and 26 cycles, respectively, so that for specimen 2 the last strain was measured after 20 cycles and for specimen 3 after 25 cycles. Unfortunately it was not possible in these experiments to make sufficiently accurate measurements of the change in thickness of the specimens (which according to the first variant of the theory should remain constant and which according to the second variant of the theory should increase in the same proportion as the length and diameter). Thus the experiments do allow us to make a comarative assessment of criteria (1.1) and (1.2).

It is seen that qualitatively the theory is borne out by experiment. The maximum values of the additional extensions of the specimens (the strains which are accopmpanied by a plastic change in volume) are about 0.4% , i.e. are comparable with the width of the plastic hysteresis loop and thus are quantities of the basic order. Experimental studies in this field are continuing and the results will be published in the near future.

Concerning plastic cavitation and its probable effect on the cyclic strength of materials, frequent mention is to be found in the revelant literature. It is considered to be a process mainly of a granular nature which initiates the formation of fatigue cracks. The latter are considered to be the result of the merging of a series of body defects (lacunae) which occurs after cavitation is sufficiently developed. Some valuable information which supports this conclusion and which is based on the analysis of numerous visual observations using a large Zeiss microscope can be found in [16]. Nadai ([17], page 261) mentions the plastic increase in volume discovered experimentally under conditions of large deformations. Some authors have emphasized the impossibility of identifying cavitation with the growth of existing cracks [18]. However, until now "the mechnism of plastic cavitation under cyclic loading has not been clarified" ([19], page 437). It has been shown above (at least for quasi-static isothermal deformations) that plastic cavitation follows from the associated flow law and the assumption that the loading condition, even very slight, depends on σ_n or on the mean normal stress. Within the framework of these assumptions four variants of the phenomenological theory of plastic cavitation have been derived.

In conclusion we raise the delicate question, which quite likely has already occureed to the reader, as to what extent the above theory agrees with modern concepts in solid-state physics and particularly with the concepts of dislocations and the part they play in plastic deformation. Is not the concept of a solid body as a granular medium with a high cohesion between particles an archaism similar to the concept of shear as the sliding of one plane over another in the presence of dry friction? For it has been proved that shear is the result of the displacement of dislocations – a process which develops gradually at a finite rate and not one which simultaneously embraces the whole plane of shear. However, it seems that both points of view can be reconciled. We should not forget that in actual bodies there are not single dislocations but many, and the properties of statistical ensembles frequently differ considerably from the individual properties of their members. We might recall the law by which a gas enclosed in some container exerts a uniform normal static pressure on the walls. Strictly speaking, this assertion is invalid since in actual fact what is called here a "uniform static pressure" is a result of the impacts of particles of the gas on the walls, i.e. the sum of discrete reactions. However, since the number of particles is very large and the time between impacts very small, in a macroscopic observation all their effects merge and as a result the concept of a gas in a container as a continuous medium at rest but attempting to expand in all directions is legitimate.

To derive the laws of plastic deformation from the properties of an individual dislocation or from a small number of dislocations would be just as wrong as deriving the properties of gases by considering the motion of one or several molecules.

When a number of dislocations are simultaneously displaced in a body and, encountering various forms of obstacles, form walls or lattices it is probable that the body is subdivided into volumetric elements relatively clear of defects separated from one another by surfaces made up of dislocations. At the same time a solid crystalline body is made up of rigid elements which are able to move one relative to another due to the surrounding defects. Is not this situation reminiscent of granular bodies? The ability of dislocations to multiply and to disappear, which complicates the picture, does not alter it qualitatively.

These arguments should not be taken as condemnation of the simple mechanical models so often used in the theory of plasticity. The final judgement on these can only come from a statistical theory of solid bodies which takes into account defects in their structure, but unfortunately this theory is still in a rudimentary state. In any case, such mechanical models are in fact substitutes for the statistical theory of solid bodies with their defects taken into account, in the same way that the Boyle-Mariott law was once a phenomenological substitute for the theory of gases – until it was derived from this theory and became its corollary. Probably the fate of the phenomenological theory of plasticity will be the same – sooner or later its basic results (and also those described in this paper) will be drived from the statistical theory of solid bodies.

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